Quantum Tera-Hertz electrodynamics in Layered Superconductors

Sergey Savel'ev,[1,2] A.L. Rakhmanov,[1,3] and Franco Nori[1,4]

[1]Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama, 351-0198, Japan
[2]Department of Physics, Loughborough University, Loughborough LE11 3TU, United Kingdom
[3]Institute for Theoretical and Applied Electrodynamics RAS, 125412 Moscow, Russia
[4]Department of Physics and MCTP, University of Michigan, Ann Arbor, MI 48109-1040, USA

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In close analogy to quantum electrodynamics, we derive a quantum field theory of Josephson plasma waves (JPWs) in layered superconductors (LSCs), which describes two types of interacting JPW bosonic quanta: one massive and the other almost-massless. We also calculate the amplitude of their decay and scattering. We propose a mechanism of enhancement of macroscopic quantum tunneling (MQT) in stacks of intrinsic Josephson-junctions (SIJJs). Due to the long-range interactions between many junctions in the LSCs, the calculated MQT escape rate Γ has a very nonlinear dependence on the number of junctions in the stack. This allows to quantitatively describe striking recent experiments in Bi2212 stacks.

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The recent surge of interest on Stacks of Intrinsic Josephson Junctions (SIJJs) is partly motivated by the desire to develop THz devices, including emitters [1, 2], filters [3], detectors [4], and nonlinear devices [5]. Macroscopic quantum tunnelling (MQT) has been, until recently, considered to be negligible in high-T$_c$ superconductors due to the d-wave symmetry of the order parameter. Recent unexpected experimental evidence [6, 7] of MQT in layered superconductors (LSCs) could open a new avenue for the applicability of SIJJs in quantum electronics [8]. This requires a quantum theory for SIJJs capable of describing quantitatively this new stream of remarkable experimental data. In contrast to a single Josephson junction, SIJJs are strongly coupled along the direction perpendicular to the layers. This is because the thickness of these layers is of the order of a few nm, which is much smaller than the magnetic penetration length. This results in a profoundly nonlocal electrodynamics [2] that strongly affects the quantum fluctuations in SIJJs.

Using a general Lagrangian approach, we derive the quantum electrodynamics of JPWs, which describes two interacting quantum fields. We analyze the first-order Feynman diagrams for: *(i)* the decay of a quantum JPW propagating along the layers and, *(ii)* JPW-JPW scattering. Employing the quantum statistics of these plasmons, we calculate the average energy of the JPWs as a function of temperature, and find it to be much higher than for the same number of non-interacting junctions. Using this general approach, we develop a quantitative theory of the MQT in SIJJs. For example, we derive the MQT escape rate, Γ, which is strongly non-linear with respect to the number of superconducting layers, N, and changes to $\Gamma \propto N$ when N exceeds a certain critical value N_c. Our results are in a good quantitative agreement with recent very exciting experiments [7].

Quantum theory for layered superconductors.— The electrodynamics of SIJJs can be described by the Lagrangian:

$$\mathcal{L} = \sum_n \int dx \left\{ \frac{1}{2}\,\dot{\varphi_n}^2 + \frac{1}{2\gamma^2}\dot{p_n}^2 - \frac{1}{2}(\partial_x \varphi_n)^2 - \frac{1}{2}(\partial_y p_n)^2 - \frac{1}{2}p_n^2 + \cos\varphi_n + \frac{1}{2}\left(\partial_x p_n \partial_y \varphi_n + \partial_y p_n \partial_x \varphi_n\right) \right\}, \quad (1)$$

where $\varphi_n \equiv \chi_{n+1} - \chi_n - 2\pi s A_y^{(n)}/\Phi_0$ is the gauge-invariant interlayer phase difference, and $p_n \equiv (s/\lambda_{ab})\partial_x \chi_n - 2\pi\gamma s A_x^{(n)}/\Phi_0$ is the normalized superconducting momentum in the nth layer. Here, we introduce the phase χ_n of the order parameter, the interlayer distance s, the in-plane λ_{ab} and out-of-plane λ_c penetration depths, the anisotropy parameter $\gamma = \lambda_c/\lambda_{ab}$, flux quantum Φ_0, and vector potential $\vec{A}$. The in-plane coordinate x is normalized by λ_c; the time t is normalized by $1/\omega_J$, where the plasma frequency is ω_J; also, $\partial_x = \partial/\partial x$, $\partial_y f_n = \lambda_{ab}(f_{n+1} - f_n)/s$, and $\dot{} = \partial/\partial t$. We choose the z axis pointed along the magnetic field. Varying the action $\mathcal{S} = \int dt\, \mathcal{L}$ produces the dynamical equations

$$\ddot{\varphi}_n - \partial_x^2 \varphi_n + \sin\varphi_n + \partial_x\partial_y p_n = 0,$$
$$\frac{1}{\gamma^2}\ddot{p}_n - \partial_y^2 p_n + p_n + \partial_x \partial_y \varphi_n = 0, \quad (2)$$

which reduces to the usual coupled sine-Gordon equations [9] for $\gamma^2 \gg 1$. Note that a Lagrangian approach for SIJJs can be formulated only for two interacting fields φ and p, but not for φ alone. This because of the 2D nature of the vector potential in SIJJs. So particles with two types of polarization can propagate. For a 1D Josephson junction, only one polarization is enough.

Linearizing Eqs. (2) results in the spectrum $\omega^2 = 1 + k_x^2/(1 + k_y^2)$ of the classical JPWs in the continuous limit (i.e., $k_y s \ll 1$) and $\gamma^2 \gg 1$. Here, k_x and k_y are the wave vectors (momentums in the quantum description; here, $\hbar = 1$) of the JPWs. In or-

der to quantize the JPWs we introduce the Hamiltonian, $\mathcal{H} = \sum_n \int dx(\Pi_{\varphi_n}\varphi_n + \Pi_{p_n}p_n) - \mathcal{L}$, with the momenta Π_{φ_n} and Π_{p_n} of the φ_n and p_n fields, and require the standard commutation relations $[\Pi_{\varphi_n}(x), \varphi_{n'}(x')] = -i\delta(x-x')\delta_{n,n'}$, $[\Pi_{p_n}(x), p_{n'}(x')] = -i\delta(x-x')\delta_{n,n'}$ (all others commutators are zero), where δ is either a delta function or Kronecker symbol. Expanding $\cos\varphi_n = 1 - \varphi_n^2/2 + \varphi_n^4/24 - \ldots$, we can write $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\rm an}$, where we include terms up to φ_n^2 in $\mathcal{H}_0$, and the anharmonic terms in $\mathcal{H}_{\rm an}$. Diagonalizing $\mathcal{H}_0$, we obtain the Hamiltonian for the Bosonic free fields a and b: $\mathcal{H}_0 = \sum_{k_y}\int(dk_x/2\pi)\left\{\varepsilon_a(\vec{k})\, a^+a \;+\; \varepsilon_b(\vec{k})\, b^+b)\right\}$, where the energy of the quasiparticles are

$$\varepsilon_a(\vec{k}) = \left(1 + \frac{k_x^2}{1+k_y^2}\right)^{1/2}, \quad \varepsilon_b(\vec{k}) = \frac{1}{\gamma}\frac{|k_x k_y|}{\sqrt{k_y^2+1}} \qquad (3)$$

up to $1/\gamma^2$, for $\gamma \gg 1$. The energy $\varepsilon_a(\vec{k})$ coincides with the frequency $\omega(k_x, k_y)$ for *classical* JPWs, while the quantum bosonic field b corresponds to the gapless branch of the excitations. The original fields φ_n, p_n in Eq. (1) are related to the free Bosonic fields a and b by

$$\varphi \approx \frac{a^+ + a}{\sqrt{2\varepsilon_a}} + \frac{\mathcal{Z}^2}{\gamma^2}\frac{b^+ + b}{\sqrt{2\varepsilon_b}}, \quad p \approx \mathcal{Z}\left(\frac{a^+ + a}{\sqrt{2\varepsilon_a}} - \frac{b^+ + b}{\sqrt{2\varepsilon_b}}\right), \qquad (4)$$

where $\mathcal{Z} = k_x k_y/(k_y^2+1)$. The case of a *single* Josephson junction corresponds to $k_y = 0$ resulting in a *single* field, a.

Thermodynamics of quantum JPWs.— Finite temperatures, T, excite both a and b quasiparticles providing contributions of the JPWs to the thermodynamical quantities. The thermal equilibrium internal energy $E(T) = \sum_{k_y}\int dk_x/(2\pi)[\varepsilon_a\, n_a + \varepsilon_b\, n_b]$ of the system can be calculated using the usual Bosonic distributions $n_{a,b} = 1/[\exp(\varepsilon_{a,b}/T) - 1]$. The calculated dependence of $E(T)$ for SIJJs and, for comparison, for an "equivalent" stack of non-interacting Josephson junctions is shown in Fig. 1a. This clearly shows that the thermodynamic energies are significantly different for these systems, especially at low temperatures. Thus, finite temperatures easily thermally excite JPWs in layered superconductors, compared with the case of non-interacting junctions. The main origin of this enhancement is the suppression of both excitation energies $\varepsilon_{a,b}(k_x, k_y)$ when increasing k_y, which is associated with a stronger interlayer interaction. Other thermodynamic quantities, e.g., heat capacity, can be easily calculated using the standard expressions.

Interaction of Bosonic fields.— The interaction between the a and b fields, including the self-interaction, occurs due to the anharmonic terms in $\mathcal{H}_{\rm an} \approx (-1/24)\sum_n \int dx\; \varphi_n^4 + \ldots$, where φ is given in (4). Here we consider the dominant first-order perturbation terms. Using the interaction representation,we obtain the amplitude $S_{\rm if} = 2\pi i\langle f|\mathcal{H}_{\rm an}|i\rangle\delta(\varepsilon_i - \varepsilon_f)$ for the transition from the initial state $|i\rangle$ to a final state $|f\rangle$, where $\varepsilon_{i,f}$ are the energies of the initial and final states. To first-order approximation in $1/\gamma^2$, a decay (see Fig. 1b) of an a-JPW can occur in two channels: either $3a$ or $2a + b$. The amplitude, $S_{\rm decay}$, of the decay of the quantum a-JPW propagating along the x-axis, i.e., along the layers, $\vec{k}_1 = (k_{x1},\ 0)$, is determined by

$$S_{\rm decay} = \int \frac{i\prod d^2\vec{k}_l}{2^8\, 3\, \pi^5}\left\{\frac{\delta\Big(\varepsilon_a(\vec{k}_1) - \sum\varepsilon_a(\vec{k}_l)\Big)}{\sqrt{\varepsilon_a(\vec{k}_1)\prod\varepsilon_a(\vec{k}_l)}}\right. \qquad (5)$$
$$\left. + \frac{\delta\Big(\varepsilon_a(\vec{k}_1) - \varepsilon_a(\vec{k}_2) - \varepsilon_a(\vec{k}_3) - \varepsilon_b(\vec{k}_4)\Big)}{(4\gamma^2/9)\ \mathcal{Z}^2(\vec{k}_4)\sqrt{\varepsilon_a(\vec{k}_1)\varepsilon_a(\vec{k}_2)\varepsilon_a(\vec{k}_3)\varepsilon_b(\vec{k}_4)}}\right\}\delta\Big(\vec{k}_1 - \sum\vec{k}_l\Big)$$

where the sums and products are performed over the final states with momenta $\vec{k}_l$. Eq. (5) predicts the probability $|S_{\rm decay}|^2$ to create JPWs propagating perpendicular to the layers by an a-JPW quantum propagating along the layers. Using Eqs. (3) and (5), one can conclude that the amplitude $S_{\rm decay}$ diverges for large k_{yl}, when resonance conditions $[\varepsilon_a(\vec{k}_1) = 2$ or $3]$ are fulfilled. For the former case (in dimensional units, $\varepsilon_a(\vec{k}_1) = 2\hbar\omega_J$) the a-JPWs create a-b-JPW pairs, while at $\varepsilon_a = 3\hbar\omega_J$ the $2a$-excitations diverge. Indeed, due to the φ^4 nonlinear interaction, a particle can only create two more additional particles, which could be either $2a$ or $a + b$. The first process has a threshold $2\hbar\omega_J$ (similar to the $2mc^2$ rest energy threshold for $e^- + e^+$ pair creation in QED), while the second one has a $\hbar\omega_J$ energy threshold due to the gapless nature of the b particles. Figure 1d shows the calculated probability, $|S_{\rm decay}|^2$, of decay of a JPW-a-quantum versus the energy ε_a of the initial a-quantum. Both resonance peaks are clearly seen.

We can similarly analyze the scattering of a-JPWs. The diagrams in Fig. 1c show two input particles as the initial state $|i\rangle$, corresponding to particles "1" and "2", while the final state $|f\rangle$ contains free particles "3" and "4". These diagrams do not diverge for any input particle momentum. However, the scattering probability enormously increases for large transverse momentum transfer $(k_{y1} - k_{y3})$, if the energies of the initial particles are close to $\hbar\omega_J$. This can occur either for low k_x or large k_y of the particles 1 and 2. The decay and scattering resonances occur due to the unusual anisotropic spectrum of the JPWs, i.e., $\varepsilon_a(k_x, k_y \to \infty) = 1$ and $\varepsilon_b(k_x, k_y \to \infty) = k_x/\gamma$.

Enhancement of macroscopic quantum tunneling.— Now we apply our theory to interpret very recent experiments [7] on MQT in Bi2212. To observe MQT, an external current J, close to the critical value J_c, was applied [7]. This produces an additional contribution $j\varphi_n$ in the Lagrangian (1). When tunneling occurs, the phase difference in a junction changes from 0 to 2π, which can be interpreted as the tunnelling of a fluxon through the contact. This process can be safely described within a semiclassical approximation and we use

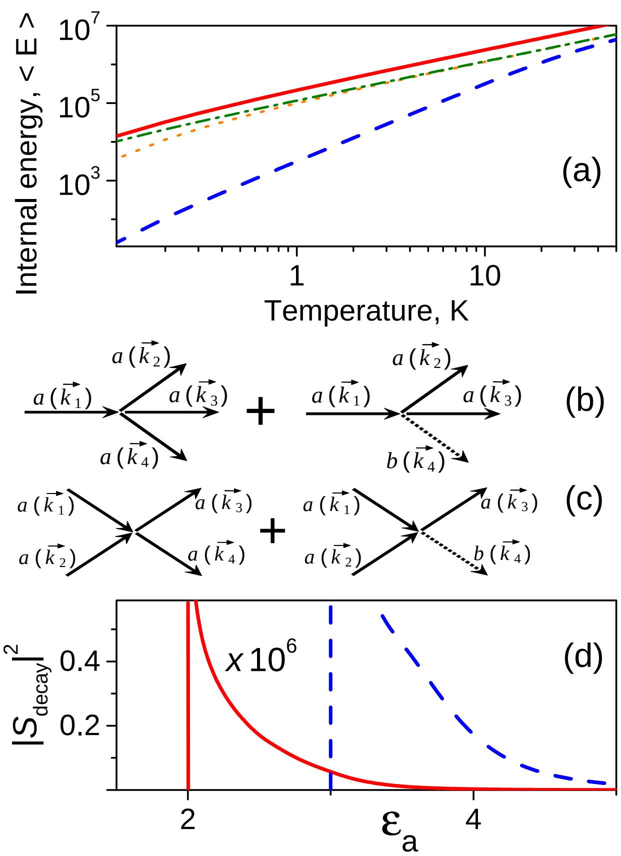

FIG. 1: (Color online)(a) Thermal energy $\langle E\rangle(T)$ of quantum JPWs versus temperature T for SIJJs (top red solid line). The energy $\langle E\rangle$ is normalized by $\hbar\omega_J$ and sample volume. For comparison, the bottom dashed blue line shows $\langle E\rangle(T)$ for the *same* number of *non*-interacting Josephson junctions and with the *same* parameters. The dashed-dotted green and the dotted orange lines correspond to the contributions of the a and b fields to the energy, respectively. The chosen parameters are standard for Bi2212: $s = 15$ Å, $\lambda_{ab} = 2000$ Å, $\gamma = 600$, and $\omega_J/2\pi = 150$ GHz. (b,c) Feynmann diagrams for decay (b) and scattering (c) of JPWs quanta due to anharmonic interactions. (d) The probability per unit time, $|S_{\rm decay}|^2$, for an a-JPWs to decay, versus energy $\varepsilon_a(\vec{k}_1)$ of the initial particle, for $\gamma = 300$. Both axes are in dimensionless units. The blue dashed line in (d) corresponds to the $a \to 3a$ channel pair-production and the red solid line to the $a \to 2a + b$ channel.

the approach developed in Refs. 7, 10 to calculate the escape rate $\Gamma = (\omega_P/2\pi)\sqrt{120\pi B}\exp(-B)$ of a fluxon through the potential barrier. Here, ω_P is the oscillation frequency of a fluxon near the effective potential minimum, and $B = \int_{-\infty}^{\infty} d\tau\ \mathcal{L}(\tau = it)$ is described by the Lagrangian (1) with the classical fields determined by Eqs. (2), if we add the term $j = J/J_c$ in the right-hand-side of the first equation. In the limit $\gamma^2 \gg 1$, the equation for φ is reduced to standard coupled sine-Gordon equations [9], which in the continuous limit, $k_y s \ll 1$ and $y = ns/\lambda_{ab}$, reads $\left(1 - \partial^2/\partial y^2\right)[\ddot{\varphi} + \sin\varphi] - \partial^2\varphi/\partial x^2 = j$. We seek a solution of the last equation in the form $\varphi = \psi(x, y, t) + \arcsin(j)$, where the field ψ obeys

$$\left(1 - \frac{\partial^2}{\partial y^2}\right)\left[\ddot{\psi} - j(1-\cos\psi) + \sqrt{1-j^2}\sin\psi\right] - \frac{\partial^2\psi}{\partial x^2} = 0. \tag{6}$$

Following the experimental setup [7], here we consider the SIJJs having the size $L \gg s$ along the y direction, i.e., the total number of contacts $N = L/s \gg 1$, and the size of the SIJJs in the x direction, $2d$, is smaller than the Josephson length, $\lambda_J = \gamma\sqrt{s\lambda_{ab}/2}$.

We can linearize Eq. (6) in all junctions except one, where the fluxon tunnels. The linearized equation can be solved by using the Fourier transformation, $\psi = \sum_m \int \exp(-i\omega t)\cos(k_{xm}x)\psi_m(y,\omega)d\omega/2\pi$, where $k_{xm} = \lambda_c\pi(2m+1)/2d$. Since in the experiment [7] the sample connects two bulk superconductors, we can choose the phase difference to be zero at the top ($y = L_1$) and bottom ($y = L_1 - L$) layers of the sample, and $y = 0$ corresponds to the position of the fluxon tunneling. As a result, we derive the solution of the linearized equations in the form $\psi_m(y) = \psi_m(0)\sinh[q_m(L_1 - y)]/\sinh[q_m L_1]$, for $y > 0$, and $\psi_m(y) = \psi_m(0)\sinh[q_m(L - L_1 + y)]/\sinh[q_m(L - L_1)]$ for $y < 0$. Here, $q_m^2 = (k_{xm}^2 + \sqrt{1-j^2} - \omega^2)/(\sqrt{1-j^2} - \omega^2)$. Following the method described in Ref. 2 and requiring the continuity of both ψ and the current flowing through the central ($y = 0$) contact, we obtain the nonlinear equation for the phase difference in the junction with $y = 0$:

$$\ddot{\psi} - j(1-\cos\psi) + \sqrt{1-j^2}\sin\psi = \frac{\lambda_J^2}{\lambda_c^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}$$
$$\times\sum_m \frac{k_{xm}^2}{q_m}\frac{\sinh(q_m L_1)\sinh(q_m(L - L_1))}{\sinh(q_m L)}\cos(k_{xm}x)\psi_m(\omega). \tag{7}$$

For an infinite ($L, d \to \infty$) sample, this equation coincides with the nonlocal equation for the Josephson vortex in the SIJJs [2]. Eq. (7) can be used to describe the tunneling of a fluxon through a SIJJs with any width d and any number of layers N. However, such a treatment can only be done numerically.

Now, we adopt Eq. (7) for the short ($d/\lambda_J \ll 1$) SIJJs used in [7], where $d/\lambda_J \approx 2\mu\mathrm{m}\,/\,5\mu\mathrm{m} = 0.4$. In this case, the phase difference φ changes slowly with x and the main contribution to the sum in the right-hand-side of (7) comes from the first harmonic $k_{xm} = \pi\lambda_c/2d$. Neglecting contributions to the tunnelling process arising from higher-frequencies, $\omega \geq \omega_J(1-j^2)^{1/4}$, and integrating Eq. (7) over dx, we derive for the phase difference $\bar{\psi}$, averaged over x, the equation: $d^2\bar{\psi}/dt^2 = -\partial V/\partial\bar{\psi}$. Here the effective potential $V(\bar{\psi})$ can be written as

$$V(\bar{\psi}) = j(\sin\bar{\psi} - \bar{\psi}) - \sqrt{1-j^2}(\cos\bar{\psi} - 1) - g_n(j)\frac{\bar{\psi}^2}{2}\ , \tag{8}$$

where $n = L_1\lambda_{ab}/s$ labels the contact through which the

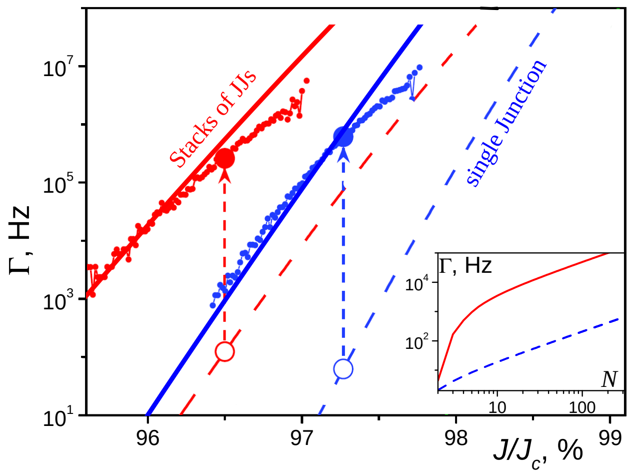

FIG. 2: (Color online) The MQT escape rate Γ versus dimensionless external current j. Red and blue points are the experimental data, from Ref. 7, for two different samples. Red and blue dashed lines are the curves $\Gamma_0(j)$, Eq. (11), for these samples taken from [7]. Red and blue solid lines are the functions $\Gamma(j)$ calculated from Eq. (10), using data from Table 1 of [7], for their samples US1 and US4, $\gamma = 600$, $s = 15$ Å. The inset shows the dependence of the escape rate Γ versus the number of contacts N in the SIJJs, using parameters for sample US4 of [7] at $j = 0.97$ (red solid line). The blue dashed line shows $N\Gamma_0$.

fluxon tunnels,

$$g_n(j) = \frac{2(1-j^2)^{1/2}\,Q}{\pi}\,\frac{\sinh(Qn)\sinh[Q(N-n)]}{\sinh(QN)}, \quad (9)$$

and $Q(j) = \pi\gamma s\left(1-j^2\right)^{1/4}/2d$. For an applied current J close to J_c, where tunnelling was observed [7], we can expand both $\cos\bar{\psi}$ and $\sin\bar{\psi}$ and, finally, derive $V(\bar{\psi}) = -\bar{\psi}^2(\bar{\psi}-\psi_1)/6$, where $\psi_1(j) = 3[\sqrt{1-j^2} - g_n(j)]$. Using a semiclassical approach [10] (i.e., $B = \int_0^{\psi_1}[2V(\bar{\psi})]^{1/2}d\bar{\psi}$), and taking into account that the fluxon can tunnel through any junction of the SIJJs, we derive (now in dimensional units)

$$\frac{\Gamma}{\Gamma_0} = \sum_{n=0}^{N}(1-g_n)^{5/4}\exp\left\{-\frac{36\,U_0}{5\,\hbar\omega_P}\left[(1-g_n)^{5/2}-1\right]\right\}, \quad (10)$$

where the summation is taken over all N contacts. Here, the effective Josephson frequency is $\omega_P(j) = \omega_J(1-j^2)^{1/4}$, the height of the potential barrier $U_0 = 2E_J(1-j^2)^{3/2}/3$, the Josephson energy $E_J = \Phi_0 J_c/2\pi c$, and the escape rate $\Gamma_0(j)$ for a single Josephson junction (see, e.g., [7]) is given by

$$\Gamma_0(j) = \frac{6\,\omega_P(j)}{\pi}\sqrt{\frac{6\pi U_0(j)}{\hbar\omega_P(j)}}\exp\left(-\frac{36\,U_0(j)}{5\,\hbar\omega_P(j)}\right), \quad (11)$$

Figure 2 shows $\Gamma(j)$, which very well describes experimental results in [7]. Some deviation between the experimental data and the theoretical prediction at high currents is due to a significant lowering of the potential barrier resulting in a decrease of the accuracy of the semiclassical approximation. The dependence of Γ on the number N of junctions is nonlinear due to the long-range interaction between different junctions, described by the last term in the expression (8) for the effective potential. This nonlinearity is strong for relatively small $N \lesssim N_c = d/\gamma L$ and the escape rate becomes proportional to N when the SIJJs thickness L exceeds the effective interaction length d/γ. Very different types of MQT models in SIJJs, with no quantitative comparison with experimental data, are also being studied in [12]. For instance, here we consider the inductive coupling among layers, which is known to be strong, instead of the weak capacitive coupling among layers used in [12].

Conclusions.— We analyze the quantum effects in SIJJs. We develop a model for quantum excitations in SIJJs using two Bosonic fields. We also describe the interactions and thermodynamics of these fields. Moreover, we suggest a semiclassical theory of the fluxon quantum tunneling in SIJJs, which is in good agreement with recent remarkable experimental observations. The obtained results might be potentially useful for future designs of quantum THz devices.

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[1] R. Kleiner *et al.*, Phys. Rev. Lett. **68**, 2394 (1992); G. Hechtfischer *et al.*, Phys. Rev. Lett. **79**, 1365 (1997); E. Goldobin *et al.*, Phys. Rev. B **57**, 130 (1998); J. Zitzmann *et al.*, Phys. Rev. B **66**, 064527 (2002); M. Tachiki *et al.*, Phys. Rev. B **71**, 134515 (2005); A. A. Abdumalikov *et al.*, Phys. Rev. B **72**, 144526 (2005).

[2] S. Savel'ev *et al.*, Phys. Rev. B **72**, (2005); Physica C **437-438** 281 (2006).

[3] S. Savel'ev *et al.*, Phys. Rev. Lett. **94**, 157004 (2005); H. Susanto *et al.*, Phys. Rev. B **71**, 174510 (2005);

[4] S. Savel'ev *et al.*, Phys. Rev. Lett. **95**, 187002 (2005).

[5] S. Savel'ev *et al.*, Nature Physics **2**, No 8, in press (2006).

[6] T. Bauch *et al.*, Phys. Rev. Lett. **94**, 087003 (2005); T. Bauch *et al.*, Science **311**, 57 (2006).

[7] X. Y. Jin *et al.*, Phys. Rev. Lett. **96**, 177003 (2006).

[8] J.Q. You, F. Nori, Phys. Today **58** (11), 42 (2005); R. McDermott *et al.*, Science **307**, 1299 (2005); A. J. Berkley *et al.*, Science **300**, 1548 (2003).

[9] L. N. Bulaevskii *et al.*, Phys. Rev. B **50**, 12831 (1994).

[10] S. Coleman, Phys. Rev. D **15**, 2929 (1977).

[11] H. Grabert, P. Olschowski, and U. Weiss, Phys. Rev. B **36**, 1931 (1987).

[12] M. Machida, T. Koyama, cond-mat/0605404, preprint;

M.V. Fistul, cond-mat/0606751.